STUDENTS

National Student Team Contest (first stage)
Solution of task 8. Colored nanofilms

The color of the film is governed by interference maximum condition:

$$\lambda = 2Dn \tag{1}$$

Since thickness of the film wasn't changed, we get an expression by dividing eq. (1) written for both cases:

$$\frac{\lambda_{green}}{\lambda_{violet}} = \frac{n_{green}}{n_{violet}} = \frac{530}{430} = 1.23 \tag{2}$$

Then we need to use Bruggeman effective media approximation. According to it one can obtain for two component system:

$$(1-P)\frac{\epsilon_{Si}-\epsilon_{eff}}{\epsilon_{Si}+2\epsilon_{eff}} + P\frac{1-\epsilon_{eff}}{1+2\epsilon_{eff}} = 0 \tag{3}$$

$$(P-1)(\epsilon_{Si}-\epsilon_{eff})(1+2\epsilon_{eff}) = P(1-\epsilon_{eff})(\epsilon_{Si}+2\epsilon_{eff}) \tag{4}$$

$$(P-1)(\epsilon_{Si}-\epsilon_{eff}+2\epsilon_{Si}\epsilon_{eff}-2\epsilon_{eff}^2) = P(\epsilon_{Si}-\epsilon_{eff}\epsilon_{Si}+2\epsilon_{eff}-2\epsilon_{eff}^2) \tag{5}$$

$$P(\epsilon_{Si}-\epsilon_{eff}+2\epsilon_{Si}\epsilon_{eff}-2\epsilon_{eff}^2) - (\epsilon_{Si}-\epsilon_{eff}+2\epsilon_{Si}\epsilon_{eff}-2\epsilon_{eff}^2) = P(\epsilon_{Si}-\epsilon_{eff}\epsilon_{Si}+2\epsilon_{eff}-2\epsilon_{eff}^2) \tag{6}$$

$$\epsilon_{Si}-\epsilon_{eff}+2\epsilon_{Si}\epsilon_{eff}-2\epsilon_{eff}^2 = P(3\epsilon_{eff}\epsilon_{Si}-3\epsilon_{eff}) \tag{7}$$

$$2\epsilon_{eff}^2+\epsilon_{eff}\cdot(3P(\epsilon_{Si}-1)+1-2\epsilon_{Si})-\epsilon_{Si}=0 \tag{8}$$

or

$$P = \frac{\epsilon_{Si}-\epsilon_{eff}+2\epsilon_{Si}\epsilon_{eff}-2\epsilon_{eff}^2}{3\epsilon_{eff}(\epsilon_{Si}-1)} \tag{9}$$

One can obtain for first case of green film:

$$2\epsilon_{eff}^2+\epsilon_{eff}\cdot(0.6(3\cdot 2.1)+1-2\cdot 3.1)-3.1=0 \tag{10}$$

or

$$2\epsilon_{eff}^2-1.42\cdot\epsilon_{eff}-3.1=0 \tag{11}$$

then

$$\epsilon_{eff} = \frac{1.42\pm\sqrt{1.42^2+4\cdot 2\cdot 3.1}}{4} = 1.65 \tag{12}$$

then

$$n_{\text{green}}=\sqrt{\epsilon_{eff}}=1.28 \tag{13}$$

Let’s move to violet film.

$$n_{violet}=\frac{n_{\text{green}}}{1.23}=\frac{1.28}{1.23}=1.04 \tag{14}$$

$$\epsilon_{vio}=n_{violet}^{2}=1.08 \tag{15}$$

$$P=\frac{3.1-1.08+2\cdot 3.1\cdot 1.08-2\cdot 1.08^{2}}{3\cdot 1.08(3.1-1)}=\frac{2.02+6.7-2.33}{6.8}=0.94 \tag{16}$$

Therefore porosity of new film is equal to 94%.